Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

■ : explanation
∵ is 'because'
∴ is 'therefore'

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA11/01**

Mathematics

January 2022

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

1. Find

$$\int\left(\frac{8x^3}{5} - \frac{2}{3x^4} - 1\right) dx$$

giving each term in simplest form.

(4)

Integration

① Write equation in easier form for integration

$$\frac{8x^3}{5} - \frac{2}{3x^4} - 1 = \frac{8}{5}x^3 - \left(\frac{2}{3} \times \frac{1}{x^4}\right) - 1$$

$\therefore x^0 = 1$

$$\frac{8}{5}x^3 - \frac{2}{3}\frac{1}{x^4} * - 1 = \frac{8}{5}x^3 - \frac{2}{3}x^{-4} - 1x^0$$

* indicies rule : $\frac{a}{x^b} = ax^{-b}$

$$\therefore \frac{8}{5}x^3 - \frac{2}{3}x^{-4} - 1x^0$$

② Integrate

$$\int \frac{8}{5}x^3 - \frac{2}{3}x^{-4} - 1 \, dx = \left[\left(\frac{8/5}{3+1}x^{3+1}\right) + \left(\frac{-2/3}{-4+1}x^{-4+1}\right) + \left(\frac{-1}{0+1}x^{0+1}\right)\right]$$

$$= \frac{2}{5}x^4 + \frac{2}{9}x^{-3} - 1x^1 + c$$

→ DON'T FORGET!! or will lose marks

$$\therefore \frac{2}{5}x^4 + \frac{2}{9}x^{-3} - x + c$$

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Question 1 continued

Q1

(Total 4 marks)

2. $$f(x) = 11 - 4x - 2x^2$$

(a) Express f(x) in the form

$$a + b(x + c)^2$$

where a, b and c are integers to be found.

(3)

(b) Sketch the graph of the curve C with equation $y = f(x)$, showing clearly the coordinates of the point where the curve crosses the y-axis.

(2)

(c) Write down the equation of the line of symmetry of C.

(1)

a) Completing the square:

if $y = x^2 + bx + c$

$y = (x + \frac{b}{2})^2 + c - (\frac{b}{2})^2$

$f(x) = 11 - 4x - 2x^2$

$f(x) = -2(x^2 + 2x - \frac{11}{2})$

Completing the square: $f(x) = -2\left((x + \frac{2}{2})^2 + (-\frac{11}{2}) - (\frac{2}{2})^2\right)$

$= -2\left((x+1)^2 - \frac{11}{2} - 1\right)$

$= -2\left((x+1)^2 - \frac{13}{2}\right)$

Expanding brackets:

$f(x) = -2(x+1)^2 + 13$

$\therefore f(x) = 13 - 2(x+1)^2$ $a = 13$ $b = -2$ $c = 1$

b) To draw graph: $-2(x+1)^2 + 13$

indicates quadratic graph: $\cup$ or $\cap$

negative $\therefore$ shape of quadratic graph: $\cap$ "sad face"

where curve crosses y axis: when $x = 0$ $f(0) = -2(0+1)^2 + 13 = 11$ $\therefore (0, 11)$

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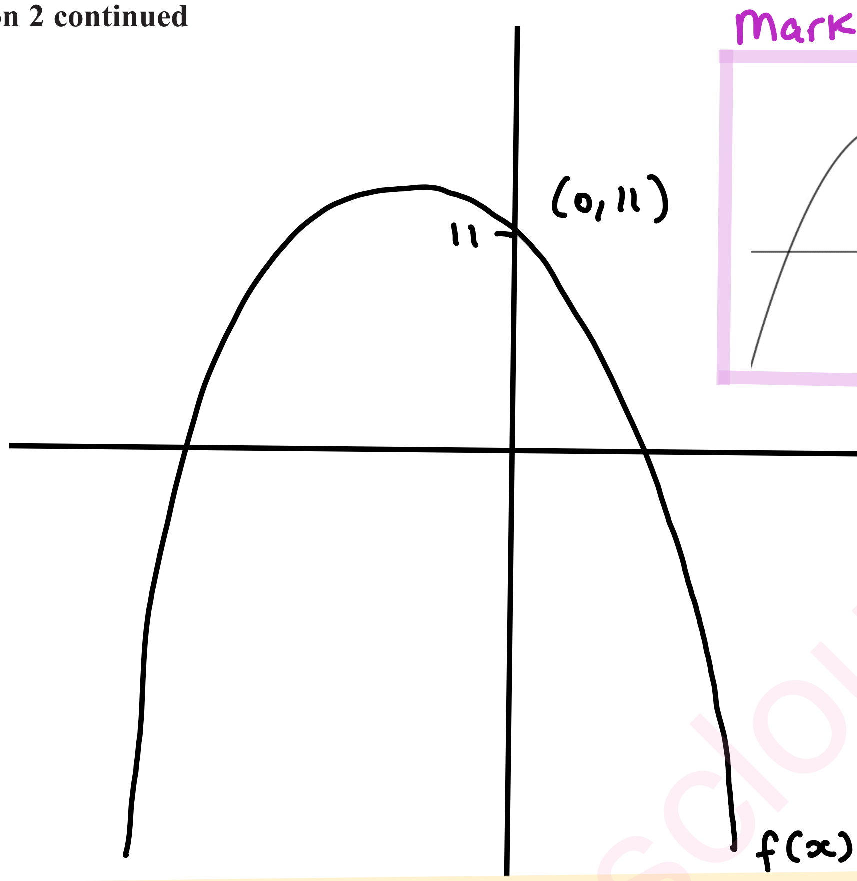
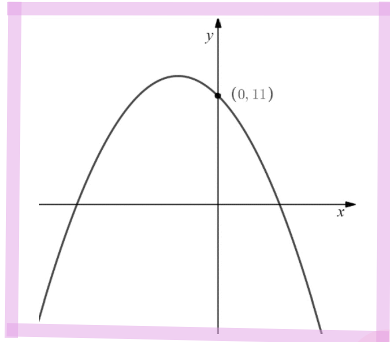
Question 2 continued



Mark scheme :



To find coordinates of maximum point:

$y = (x + \underbrace{\tfrac{b}{2}}_{\text{inverse is } x\text{-coordinate}})^2 + \underbrace{c - (\tfrac{b}{2})^2}_{y\text{-coordinate}}$

Explanation: when $y = c - (b/2)^2$: $c - (b/2)^2 = (x + b/2)^2 + c - (b/2)^2$

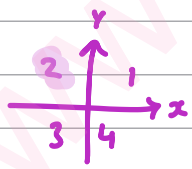
$0 = (x + b/2)^2 \quad \therefore x = -b/2$

when $x = -b/2$: $y = (-b/2 + b/2)^2 + c - (b/2)^2$

$\therefore y = c - (b/2)^2$

$-2(x+1)^2 + 13$

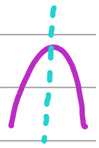
$x = -1 \qquad y = 13 \qquad (-1, 13)$



← maximum is in quadrant 2

$\therefore$ Shape of curve :

c) line of symmetry is x-coordinate of maximum :



Maximum $(-1, 13)$ $\therefore$ equation of line of symmetry is $x = -1$

Q2

(Total 6 marks)

Turn over

P 7 0 4 8 2 A 0 5 3 2

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i)

$$f(x) = (x + \sqrt{2})^2 + (3x - 5\sqrt{8})^2$$

Express f(x) in the form $ax^2 + bx\sqrt{2} + c$ where a, b and c are integers to be found.

(3)

(ii) Solve the equation

$$\sqrt{3}(4y - 3\sqrt{3}) = 5y + \sqrt{3}$$

giving your answer in the form $p + q\sqrt{3}$ where p and q are simplified fractions to be found.

(4)

i) expanding brackets

$f(x) = (x+\sqrt{2})^2 + (3x-5\sqrt{8})^2$

$= (x+\sqrt{2})(x+\sqrt{2}) + (3x-5\sqrt{8})(3x-5\sqrt{8})$

$= (x^2 + x\sqrt{2} + x\sqrt{2} + 2) + (3x-5\sqrt{8})(3x-5\sqrt{8})$

$= (x^2 + x\sqrt{2} + x\sqrt{2} + 2) + (9x^2 - 15x\sqrt{8} - 15x\sqrt{8} + 200)$

$= x^2 + x\sqrt{2} + x\sqrt{2} + 2 + 9x^2 - 15x\sqrt{8} - 15x\sqrt{8} + 200$

$= (x^2 + 9x^2) + (x\sqrt{2} + x\sqrt{2} - 15x\sqrt{8} - 15x\sqrt{8}) + (2 + 200)$

$= 10x^2 + (x\sqrt{2} + x\sqrt{2} - 15x\sqrt{4\times2} - 15x\sqrt{4\times2}) + 202$

$= 10x^2 + (x\sqrt{2} + x\sqrt{2} - 15x(2)\sqrt{2} - 15x(2)\sqrt{2}) + 202$

$= 10x^2 + (x\sqrt{2} + x\sqrt{2} - 30x\sqrt{2} - 30x\sqrt{2}) + 202$

$= 10x^2 + (2x\sqrt{2} - 60x\sqrt{2}) + 202$

$= 10x^2 - 58x\sqrt{2} + 202$

$\therefore f(x) = 10x^2 - 58x\sqrt{2} + 202$ $\quad a = 10 \quad b = -58 \quad c = 202$

ii) $\sqrt{3}(4y - 3\sqrt{3}) = 5y + \sqrt{3}$

Expand brackets: $4\sqrt{3}y - 3(\sqrt{3}\times\sqrt{3}) = 5y + \sqrt{3}$

$4\sqrt{3}y - 3(3) = 5y + \sqrt{3}$

$4\sqrt{3}y - 9 = 5y + \sqrt{3}$

P 7 0 4 8 2 A 0 6 3 2

Question 3 continued

rearrange to make y the subject & solve for y:

$$4\sqrt{3}y - 9 = 5y + \sqrt{3} \quad -5y$$
$$4\sqrt{3}y - 9 - 5y = \sqrt{3} \quad +9$$
$$4\sqrt{3}y - 5y = 9 + \sqrt{3}$$

$$(4\sqrt{3} - 5)y = 9 + \sqrt{3} \quad \div(4\sqrt{3} - 5)$$
$$y = \frac{9+\sqrt{3}}{4\sqrt{3}-5}$$

Rationalise the denominator:

$$y = \frac{9+\sqrt{3}}{4\sqrt{3}-5} \times \frac{(4\sqrt{3}+5)}{(4\sqrt{3}+5)} = \frac{(9+\sqrt{3})(4\sqrt{3}+5)}{(4\sqrt{3}-5)(4\sqrt{3}+5)}$$

$$= \frac{(9\times4\sqrt{3}) + (9\times5) + (\sqrt{3}\times4\sqrt{3}) + (\sqrt{3}\times5)}{(4\sqrt{3}\times4\sqrt{3}) + (4\sqrt{3}\times5) + (-5\times4\sqrt{3}) + (5\times-5)}$$

$$= \frac{36\sqrt{3} + 45 + 12 + 5\sqrt{3}}{48 + 20\sqrt{3} - 20\sqrt{3} - 25} = \frac{36\sqrt{3}+45+12+5\sqrt{3}}{48-25}$$

$$= \frac{41\sqrt{3}+57}{23} = \frac{41}{23}\sqrt{3} + \frac{57}{23}$$

write in form $p + q\sqrt{3}$

$$\therefore \frac{57}{23} + \frac{41}{23}\sqrt{3}$$

$$p = \frac{57}{23} \qquad q = \frac{41}{23}$$

Question 3 continued

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Question 3 continued

Q3

(Total 7 marks)

Turn over

4.

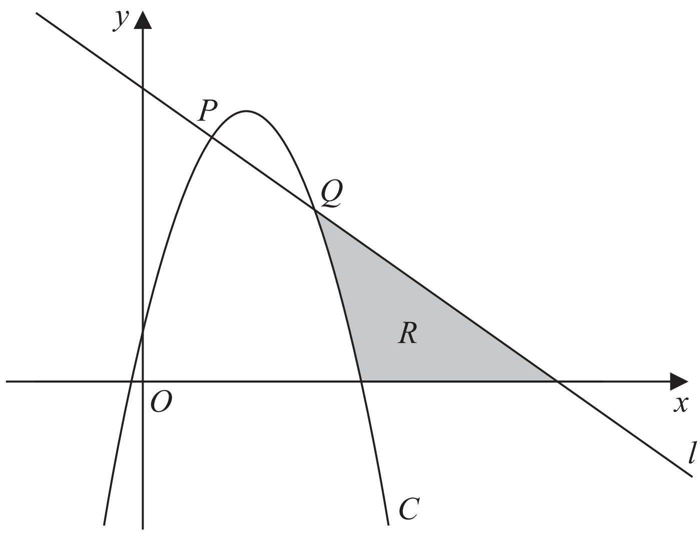


Figure 1

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 1 shows a line l with equation $x + y = 6$ and a curve C with equation $y = 6x - 2x^2 + 1$

The line l intersects the curve C at the points P and Q as shown in Figure 1.

(a) Find, using algebra, the coordinates of P and the coordinates of Q.

(4)

The region R, shown shaded in Figure 1, is bounded by C, l and the x-axis.

(b) Use inequalities to define the region R.

(3)

a) P & Q are intersections between l & C ∴ we equate them together.

① make y the subject in both equations

C : $y = 6x - 2x^2 + 1$

l : $x + y = 6 \Rightarrow y = 6 - x$

② equate the two equations

-6 $\quad 6x - 2x^2 + 1 = 6 - x \quad$ -6

$+x$ $\quad 6x - 2x^2 - 5 = -x \quad$ $+x$

$7x - 2x^2 - 5 = 0$

Question 4 continued

$$\div -1 \quad -2x^2 + 7x - 5 = 0 \quad \div -1$$
$$2x^2 - 7x + 5 = 0$$

3 Solve for x

$$(2x-5)(x-1) = 0$$

$+5$ $\quad 2x - 5 = 0 \quad +5$
$\quad 2x = 5$
$\div 2 \quad x = 5/2 \quad \div 2$

$+1 \quad x - 1 = 0 \quad +1$
$\quad x = 1$

$\therefore x_1 = 5/2 \qquad \therefore x_2 = 1$

4 find values of y by substituting x into equation of l or C.

$$y = 6 - x$$

$y_1 = 6 - x_1 = 6 - 5/2 = 7/2 \qquad \therefore y_1 = 7/2$
$y_2 = 6 - x_2 = 6 - 1 = 5 \qquad \therefore y_2 = 5$

5 $(5/2, 7/2)$ & $(1, 5)$

P coordinate comes before Q on x-axis so has smaller x-coordinate.

$$5/2 > 1$$

$\therefore P(1, 5) \qquad Q(5/2, 7/2)$

also written as $(2.5, 3.5)$

Q4

(Total 7 marks)

Turn over

P 7 0 4 8 2 A 0 1 1 3 2

b) Four inequalities to be identified

① First inequality:

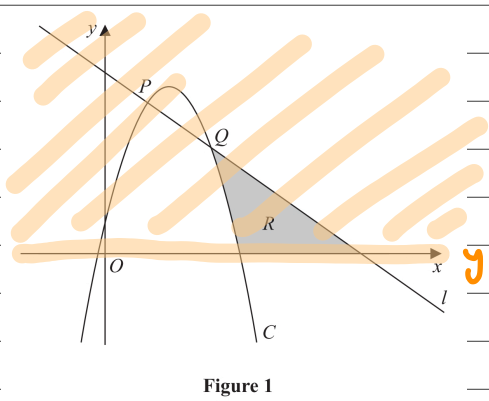


Figure 1

$y=0$

Shaded region R is in positive y-axis

$\therefore\ y \geqslant 0$ *

$\geqslant$ and NOT $>$ $\because$ Solid lines & not dashed [Solid   dashed]

② Second inequality with line l

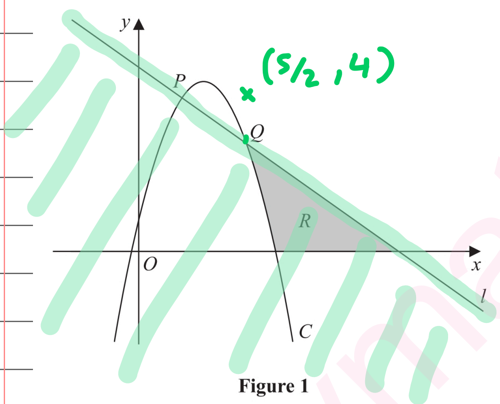


Figure 1

$y = 6 - x$

To find inequality, choose point OUTSIDE valid region, and make the inequality FALSE.

Point: $(5/2, 4)$

$4 = 6 - 5/2$

$4 = 7/2$

To make it FALSE: $4 \leq 7/2$

$\therefore\ y \leq 6 - x$ *

③ Third Inequality with Curve C

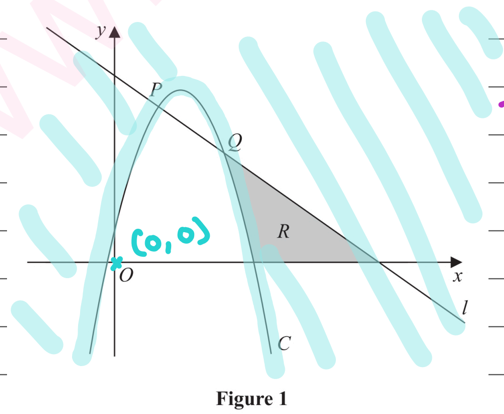


Figure 1

$y = 6x - 2x^2 + 1$

To find inequality, choose point OUTSIDE valid region, and make the inequality FALSE.

Point: $(0, 0)$

$0 = 6(0) - 2(0)^2 + 1$

$0 = 1$

To make it FALSE: $0 \geqslant 1$

$\therefore\ y \geqslant 6x - 2x^2 + 1$ *

(4) Fourth Inequality

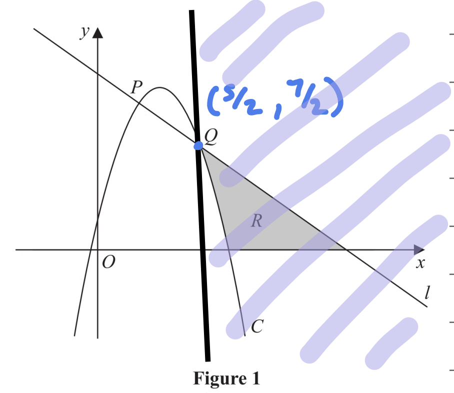


Figure 1

$x = 5/2$

$x = 5/2$

$\therefore\ x \geq 5/2$

ANSWER:

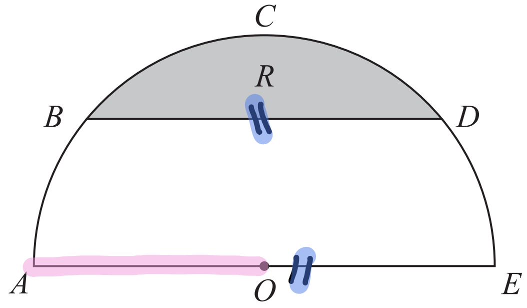
$\therefore$ region R is defined by inequalities:

$y \geq 0$, $x \geq 5/2$, $y \leq 6 - x$, $y \geq 6x - 2x^2 + 1$

$x \geq 5/2$ can be written as $x \geq 2.5$

$y \leq 6 - x$ can be written as $x + y \leq 6$

5.



Not to scale

Figure 2

Figure 2 shows a plan view of a semicircular garden *ABCDEOA*

The semicircle has

- centre O
- diameter AOE
- radius 3 m

UNITS !!

The straight line BD is parallel to AE and angle BOA is 0.7 radians.

(a) Show that, to 4 significant figures, angle BOD is 1.742 radians.

(1)

The flowerbed R, shown shaded in Figure 2, is bounded by BD and the arc BCD.

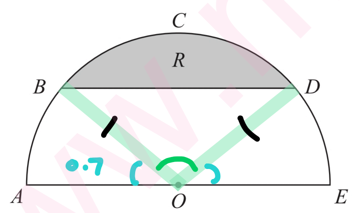
(b) Find the area of the flowerbed, giving your answer in square metres to one decimal place.

(3)

(c) Find the perimeter of the flowerbed, giving your answer in metres to one decimal place.

(3)

a)

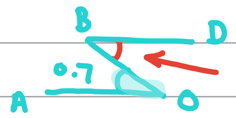


$\angle BOA = 0.7$ rad

$\angle BOA + \angle BOD + \angle DOE = \pi$ rad $(180°)$

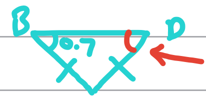
$\angle DOE = \angle BOA$

Explanation :

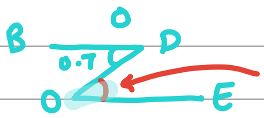
① 

Alternate angle rule $\Rightarrow$  angles are the same

$\therefore \angle DBO = 0.7$ rad

② 

BDO is isoceles triangle $\therefore \angle BDO = 0.7$ rad

③ 

alternate angle rule $\therefore \angle DOE = 0.7$ rad

P 7 0 4 8 2 A 0 1 2 3 2

Question 5 continued

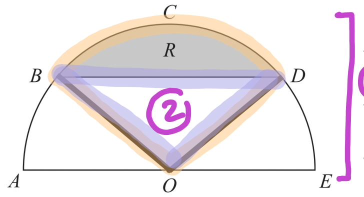
$\therefore \angle BOD = \pi - \angle BOA - \angle DOE = \pi - 0.7 - 0.7$

$= \pi - 2(0.7) = 1.741592\ldots$

$\therefore \angle BOD = 1.742$ (4 sig fig)

b) To find area of R :

Area of Sector OBCD − Area of triangle BDO



① : Area of sector : $A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2} \times 3^2 \times 1.742 = 7.839$

② : Area of a triangle: $A = \frac{1}{2}r^2 \sin\theta$

$A = \frac{1}{2} \times 3^2 \times \sin 1.742 = 4.4342118\ldots$

≈ 4.434

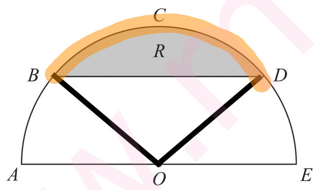
Area of R = ① − ② = $7.839 - 4.434 = 3.405$

$\therefore$ Area R $= 3.4\ m^2$ (1dp)

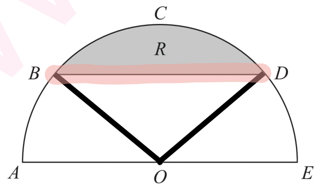
c) Perimeter of R = BCD + BD

① Find arc BCD : $S = r\theta$

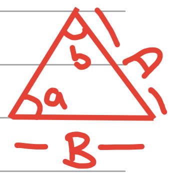
$S = 3 \times 1.742 = 5.226$

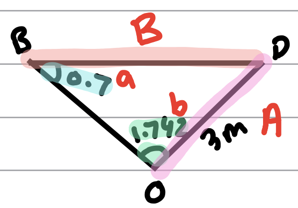


② Find length BD



use Sine rule : $\frac{\sin a}{A} = \frac{\sin b}{B}$





$\frac{\sin 0.7}{3} = \frac{\sin 1.742}{BD}$

Question 5 continued

$$\frac{\sin 0.7}{3} = \frac{\sin 1.742}{BD}$$

$\times BD$ $\quad$ $\times BD$

$$BD\left(\frac{\sin 0.7}{3}\right) = \sin 1.742$$

$\div \frac{\sin 0.7}{3}$ $\quad$ $\div \frac{\sin 0.7}{3}$

$$BD = \sin 1.742 \div \frac{\sin 0.7}{3}$$

$$BD = \underbrace{\sin 1.742 \times \frac{3}{\sin 0.7}} = \frac{3\sin 1.742}{\sin 0.7} = 4.5887303\ldots \approx 4.589$$

Dividing by fractions : $a \div \frac{b}{c} = a \times \frac{c}{b}$

③ Perimeter $= \cancel{①} + \cancel{②} = 5.226 + 4.589 = 9.815$

$\therefore$ Perimeter of $R = 9.8$ m (1dp)

Leave blank

Question 5 continued

Q5

(Total 7 marks)

P 7 0 4 8 2 A 0 1 5 3 2

Turn over

6. The curve C has equation $y = \mathrm{f}(x)$ where $x > 0$

Given that

- $\mathrm{f}'(x) = \dfrac{(x+3)^2}{x\sqrt{x}}$
- the point $P(4, 20)$ lies on C

(a) (i) find the value of the gradient at P

(ii) Hence find the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

↳ Whole numbers

(4)

(b) Find $\mathrm{f}(x)$, simplifying your answer.

(7)

a) i) We can substitute x-value of P into the gradient function (differential) $f'(x)$ to find the gradient.

$$f'(4) = \frac{(4+3)^2}{4\sqrt{4}} = \frac{7^2}{4(2)} = \frac{49}{8}$$

$\therefore$ gradient at P is $\frac{49}{8}$

ii) tangent means gradient of tangent is same as gradient of equation

① find equation of tangent using line passing through (a, b) & gradient m

equation: $(y - b) = m(x - a)$

$a = 4$
$b = 20$
$m = 49/8$

$$(y - 20) = \frac{49}{8}(x - 4)$$

② Write equation in form $ax + by + c = 0$

$$y - 20 = \frac{49}{8}(x - 4)$$

$\times 8$ $\quad 8(y - 20) = 49(x - 4)$ $\quad \times 8$

$$8y - 160 = 49x - 196$$

Question 6 continued

$$8y - 160 = 49x - 196$$

$-8y$ $\quad -8y$

$$-160 = 49x - 196 - 8y$$

$+160$ $\quad +160$

$$0 = 49x - 36 - 8y$$

$\therefore$ equation of tangent is :

$$49x - 8y - 36 = 0$$

b) $f(x) \underset{\text{integrate}}{\overset{\text{differentiate}}{\rightleftarrows}} f'(x)$

① write $f'(x)$ in easier form for integration.

$$f'(x) = \frac{(x+3)^2}{x\sqrt{x}} = \frac{(x+3)(x+3)}{x\sqrt{x}} = \frac{(x^2+3x+3x+9)}{x\sqrt{x}}$$

$$= \frac{x^2+6x+9}{x\sqrt{x}} \text{①} = \frac{x^2+6x+9}{x(x^{\frac{1}{2}})}$$

① indicies rule : $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$$= \frac{x^2+6x+9}{x^1 \times x^{\frac{1}{2}}} = \frac{x^2+6x+9}{x^{1+\frac{1}{2}}} \text{②} = \frac{x^2+6x+9}{x^{3/2}}$$

② indicies rule : $a^b \times a^c = a^{b+c}$

$$= \text{③} \frac{x^2}{x^{3/2}} + \frac{6x^1}{x^{3/2}} + \frac{9x^0}{x^{3/2}} \leftarrow \because x^0 = 1$$

$$= x^{2-3/2} + 6x^{1-3/2} + 9x^{0-3/2} = x^{\frac{1}{2}} + 6x^{-1/2} + 9x^{-3/2}$$

③ : indicies rule $\frac{a^b}{a^c} = a^{b-c}$

$$\therefore f'(x) = x^{\frac{1}{2}} + 6x^{-\frac{1}{2}} + 9x^{-\frac{3}{2}}$$

② Integrate $f'(x)$.

$$f(x) = \int f'(x)\,dx = \int 1x^{\frac{1}{2}} + 6x^{-1/2} + 9x^{-3/2}\,dx$$

$$= \left[\left(\frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1}\right) + \left(\frac{6}{-\frac{1}{2}+1} x^{-1/2+1}\right) + \left(\frac{9}{-\frac{3}{2}+1} x^{-3/2+1}\right)\right]$$

Question 6 continued

$= \frac{2}{3}x^{3/2} + 12x^{1/2} - 18x^{-1/2} + c$

③ find value of constant c by substituting $P(4, 20)$ into $f(x)$

$f(x) = \frac{2}{3}x^{3/2} + 12x^{1/2} - 18x^{-1/2} + c$

$f(4) = \frac{2}{3}(4)^{3/2} + 12(4)^{1/2} - 18(4)^{-1/2} + c = 20$

$= \frac{16}{3} + 24 - 9 + c = 20$

$= \frac{61}{3} + c = 20$

$-\frac{61}{3}$ $\quad$ $-\frac{61}{3}$

$c = -\frac{1}{3}$

$\therefore f(x) = \frac{2}{3}x^{\frac{3}{2}} + 12x^{\frac{1}{2}} - 18x^{-\frac{1}{2}} - \frac{1}{3}$

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Question 6 continued

Q6

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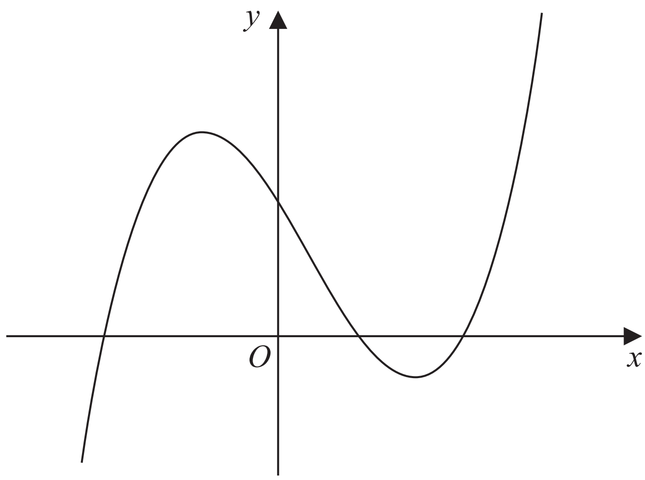

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (x + 4)(x - 2)(2x - 9)$$

Given that the curve with equation $y = \mathrm{f}(x) - p$ passes through the point with coordinates (0, 50)

(a) find the value of the constant p.
(2)

Given that the curve with equation $y = \mathrm{f}(x + q)$ passes through the origin,

(b) write down the possible values of the constant q.
(2)

(c) Find $\mathrm{f}'(x)$.
(4)

(d) Hence find the range of values of x for which the gradient of the curve with equation $y = \mathrm{f}(x)$ is less than -18
(3)

a) y-intercept of f(x) is when $x = 0$

$f(0) = (0+4)(0-2)(2(0)-9) = 72$ $\therefore (0, 72)$

translation p units down.

for $f(x) - p$, y intercept is $(0, 50)$. As $-p$ is outside f(x) brackets, it affects y-coordinates of curve only.

$72 - p = 50$

-72 $\qquad -p = -22 \qquad$ -72

$\times -1$ $\qquad p = 22 \qquad$ $\times -1$

$\therefore p = 22$

P 7 0 4 8 2 A 0 2 0 3 2

translation q units horizontally $\binom{0}{-q}$

Question 7 continued

b) $f(x+q)$ passes through origin $(0,0)$

As $+q$ is inside $f(x)$ bracket, x-coordinate only is affected so we need to look at points in $f(x)$ where $y=0$

$f(x) = (x+4)(x-2)(2x-9) = 0$

Solve for x: ① $x+4=0$, $x=-4$ ② $x-2=0$, $x=2$ ③ $2x-9=0$, $2x=9$, $x=\frac{9}{2}$

for $f(x+q)$, as q is inside brackets of $f(x)$, we do $-q$ to x values [the inverse, NOT $+q$]

$x - q = 0$

① $-4-q=0$, $q=-4$ ② $2-q=0$, $q=2$ ③ $\frac{9}{2}-q=0$, $q=9/2=4.5$

$\therefore q = -4, 2, 4.5$

c) $f'(x)$ is differential of $f(x)$.

① Expand brackets of

$$\begin{aligned} f(x) &= (x+4)(x-2)(2x-9) \\ &= (x^2+4x-2x-8)(2x-9) \\ &= (x^2+2x-8)(2x-9) \\ &= 2x^3-9x^2+4x^2-18x-16x+72 \\ &= 2x^3-5x^2-34x^1+72x^0 \end{aligned}$$

$\because x^0=1$

② differentiate

$$\begin{aligned} f'(x) &= 3(2x^{3-1}) + 2(-5x^{2-1}) + 1(-34x^{1-1}) + 0(72x^{0-1}) \\ &= 6x^2 - 10x - 34 \end{aligned}$$

$\therefore f'(x) = 6x^2 - 10x - 34$

d) $f'(x) < -18$

$6x^2-10x-34 < -18$ $\quad +18$

$6x^2-10x-16 < 0$

P 7 0 4 8 2 A 0 2 1 3 2

Question 7 continued

Solve $6x^2 - 10x - 16 = 0$ to find critical values

$$6x^2 - 10x - 16 = 0$$
$$\div 2 \qquad 3x^2 - 5x - 8 = 0 \qquad \div 2$$

factorise : $(3x - 8)(x + 1) = 0$

solve : $3x - 8 = 0$ $\quad x + 1 = 0$

$3x = 8 \qquad x = -1$

$x = \frac{8}{3}$

when x is 1 less than $-1 \Rightarrow x = -2$

$f'(-2) = 10$

$10 > -18$ but we want $f'(x) < 18$

$\therefore -1 < x$

when x is 1 more than $\frac{8}{3} \Rightarrow x = \frac{11}{3}$

$f'(\frac{11}{3}) = 10$

$10 > -18$ but we want $f'(x) < 18$

$\therefore x < \frac{8}{3}$

$\therefore$ when $f(x) < -18$, $-1 < x < \frac{8}{3}$

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Question 7 continued

(Total 11 marks)

Q7

P 7 0 4 8 2 A 0 2 3 3 2

Turn over

8. The line l_1 has equation

$$2x - 5y + 7 = 0$$

(a) Find the gradient of l_1

(1)

Given that

- the point A has coordinates $(6, -2)$
- the line l_2 passes through A and is perpendicular to l_1

(b) find the equation of l_2 giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

The lines l_1 and l_2 intersect at the point M.

(c) Using algebra and showing all your working, find the coordinates of M.

(Solutions relying on calculator technology are not acceptable.)

(3)

Given that the diagonals of a square $ABCD$ meet at M,

(d) find the coordinates of the point C.

(2)

a) gradient (m) found in form $y = mx + c$. So rearrange l_1.

l_1: $2x - 5y + 7 = 0$ $\quad +5y$

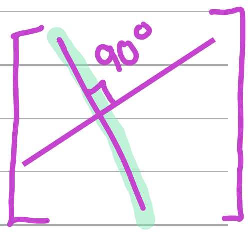
$+5y$ $\quad 2x + 7 = 5y$ $\quad \div 5$

$\div 5$ $\quad \frac{2}{5}x + \frac{7}{5} = y$

$$y = \boxed{\frac{2}{5}}x + \frac{7}{5}$$

$\therefore$ gradient $= \frac{2}{5}$

b) normal is Perpendicular to line l_1



$\therefore$ we find gradient of normal (m_n)

using Perpendicular gradient rule $m_{normal} \times m_{line} = -1$

Question 8 continued

① find gradient of normal using formula.

$$M_n \times M_{l_1} = -1$$

$$\div \tfrac{2}{5} \left(\; M_n \times \tfrac{2}{5} = -1 \; \right) \div \tfrac{2}{5}$$

$$M_n = -\tfrac{5}{2}$$

② find equation of normal by using line passing through (a, b) & gradient M

equation: $(y - b) = M(x - a)$

$a = 6$
$b = -2$
$M = -5/2$

$$(y-(-2)) = -\tfrac{5}{2}(x - 6)$$

③ Write in form $y = mx + c$

$$-2 \left(\; y + 2 = -\tfrac{5}{2}x + 15 \; \right) -2$$

$$y = -\tfrac{5}{2}x + 13$$

$$\therefore \; l_2 : \; y = -\frac{5}{2}x + 13$$

c) to find intersection M, equate equations of l_1 & l_2.

l_1: $y = \tfrac{2}{5}x + \tfrac{7}{5}$

l_2: $y = -\tfrac{5}{2}x + 13$

$$+\tfrac{5}{2}x \left(\; \tfrac{2}{5}x + \tfrac{7}{5} = -\tfrac{5}{2}x + 13 \; \right) +\tfrac{5}{2}x$$

$$-\tfrac{7}{5} \left(\; \frac{29}{10}x + \tfrac{7}{5} = 13 \; \right) -\tfrac{7}{5}$$

$$\times 10 \left(\; \tfrac{29}{10}x = \tfrac{58}{5} \; \right) \times 10$$

$$\div 29 \left(\; 29x = 116 \; \right) \div 29$$

$$x = 4$$

find y-value by substituting $x = 4$ into equation l_1 or l_2.

l_1: $y = \tfrac{2}{5}(4) + \tfrac{7}{5} = \tfrac{8}{5} + \tfrac{7}{5} = \tfrac{15}{5} = 3$

$\therefore \; y = 3$

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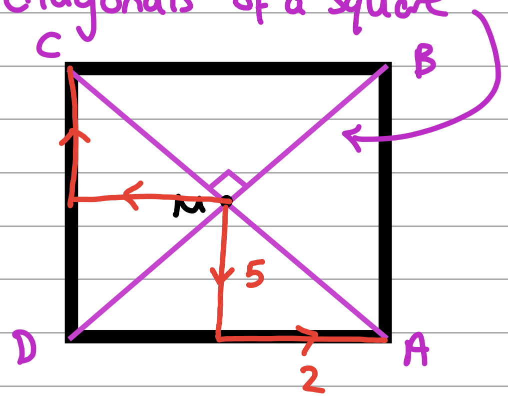
Question 8 continued

$\therefore M(4, 3)$

d) diagonals of a square



M is midpoint of AC.

$\vec{MA} = A - M = \begin{pmatrix} 6 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ 2 units right, 5 units down

$\vec{MC}$ opposite of $\vec{MA}$ $\therefore \begin{pmatrix} -2 \\ +5 \end{pmatrix}$ 2 units left, 5 units up

$C - M = \vec{MC}$

$\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$

$\therefore C(2, 8)$

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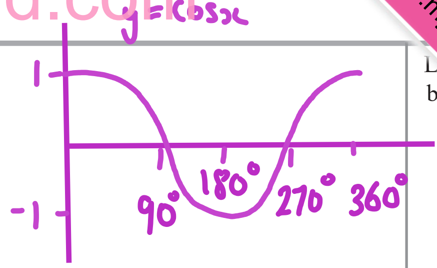
Question 8 continued

Q8

(Total 9 marks)



9.

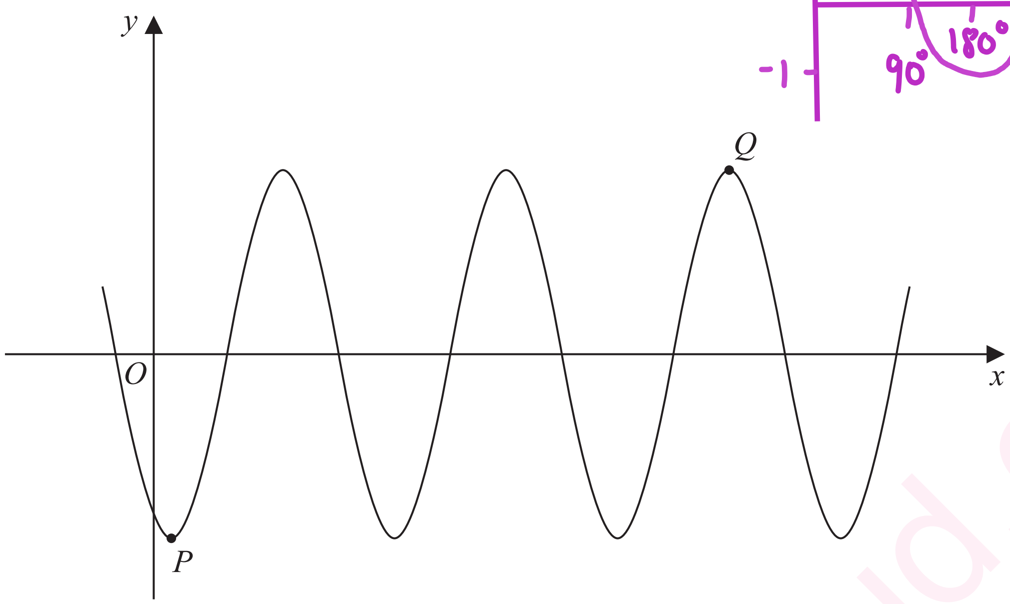


Figure 4

Figure 4 shows part of the curve with equation

$$y = A\cos(x - 30)°$$ → Unit is degrees

where A is a constant.

The point P is a minimum point on the curve and has coordinates $(30, -3)$ as shown in Figure 4.

(a) Write down the value of A.

(1)

The point Q is shown in Figure 4 and is a maximum point.

(b) Find the coordinates of Q.

(3)

a) if we consider $y = \cos x$ as $y = f(x)$, $y = A(\cos x - 30)$ can be written as $y = A f(x-30)$.
A is vertical stretch / squash of $f(x)$. As it is outside $f(x)$ brackets, it affects y-coordinates only ∴ we will only focus on y-coordinate of P, $y = -3$.

for $y = \cos x$, first turning point (∩) is a maximum $y = 1$.
for $y = A\cos(x-30)$, first turning point (∪) is a minimum $y = -3$.

$A \times 1 = -3$

$A = \frac{-3}{1}$

$\therefore A = -3$

P 7 0 4 8 2 A 0 2 8 3 2

Question 9 continued

b) Point Q is 6^{th} turning point & a maximum

① y – coordinate :

for $y = \cos x$, minimum is $y = -1$

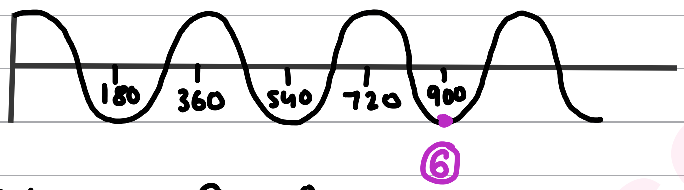
Part (a)

for $y = -3\cos(x - 30)$, maximum : $-3 \times -1 = 3$.

$y = 3$

② x coordinate :

6^{th} turning point for $y = \cos x$:



At $x = 900°$

for $y = -3\cos(x - 30)$
in form $y = -3f(x-30)$, translation of x-axis 30 units to the right $\binom{30}{0}$ ∴ –30 is inside f(x) brackets so x-coordinates only affected & we do inverse '–30' in brackets mean +30 to x-values.

$x = 900 + 30 = 930°$

∴ $Q\,(930°, 3)$

(Total 4 marks)

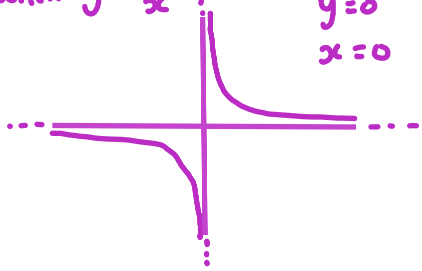
Q9

10. The curve C has equation

$$y = \frac{1}{x^2} - 9$$

when $y = \frac{1}{x}$:

Asymptotes:
$y = 0$
$x = 0$



(a) Sketch the graph of C.

On your sketch

- show the coordinates of any points of intersection with the coordinate axes
- state clearly the equations of any asymptotes

(4)

when $x = 0$ $\quad y = \frac{1}{0^2} - 9 =$ UNDEFINED
↳ not possible to divide by zero

The curve D has equation $y = kx^2$ where k is a constant.

Given that C meets D at 4 distinct points,

when $y = 0$ $\quad 0 = \frac{1}{x^2} - 9$

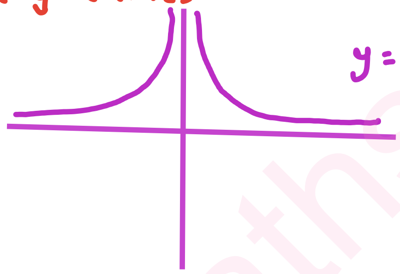
(b) find the range of possible values for k.

$x^2 = \frac{1}{9}$ $\quad \therefore \ x = \pm \frac{1}{3}$

(5)

$y = \frac{1}{x^2}$ ↙ x^2 means can't have negative y-values

$\therefore$

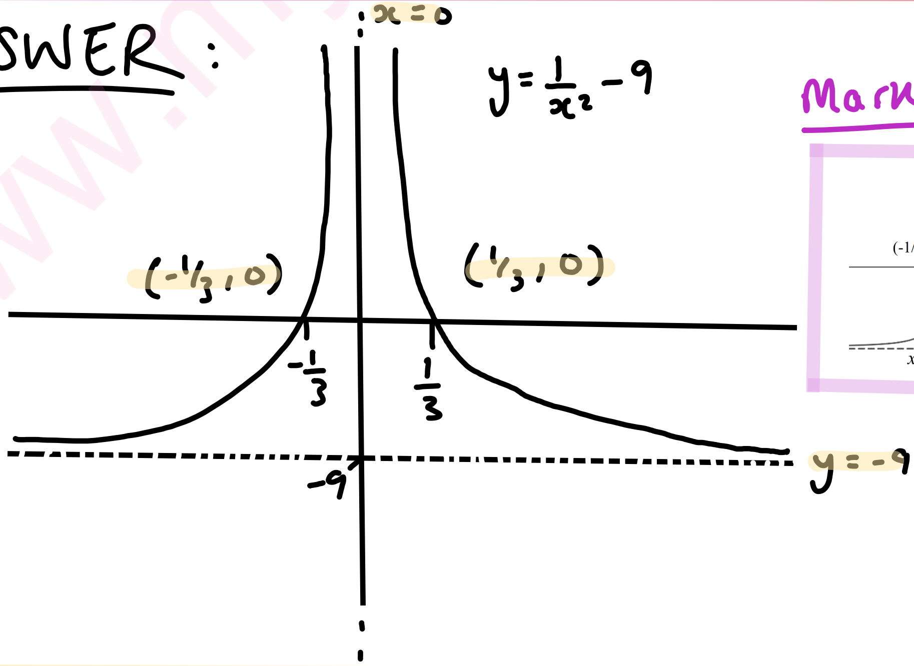


$y = \frac{1}{x^2} - 9$ ← translation 9 units down $\begin{pmatrix} 0 \\ -9 \end{pmatrix}$ of $y = \frac{1}{x^2}$

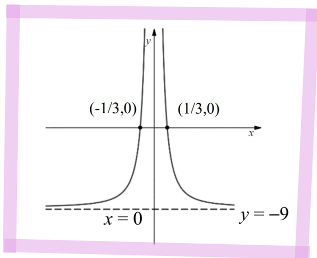
$\boxed{y = f(x) - 9}$

So y asymptote become $y = 0 - 9$
$y = -9$

ANSWER :



Mark scheme :



Question 10 continued

b) $y = Kx^2$

4 real root ∴ use discriminant rule $b^2 - 4ac > 0$

① equate curve C & curve D.

C: $y = \frac{1}{x^2} - 9$

D: $y = Kx^2$

$$\frac{1}{x^2} - 9 = Kx^2$$

$\times x^2$ $$1 - 9x^2 = Kx^4$$ $\times x^2$

$+9x^2$ $$1 = Kx^4 + 9x^2$$ $+9x^2$

-1 $$0 = Kx^4 + 9x^2 - 1$$ -1

$$Kx^4 + 9x^2 - 1 = 0$$

② 4 real roots means that when the equation of the graph is $ax^4 + bx^2 + c = 0$ discriminant is $b^2 - 4ac > 0$

$$\underset{a}{K}x^4 + \underset{b}{9}x^2 \underset{c}{-1} = 0$$

$$b^2 - 4ac > 0$$

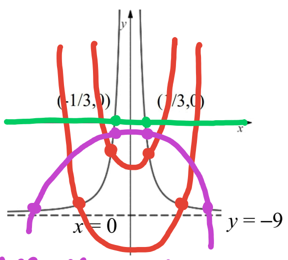
$$9^2 - 4(K)(-1) > 0$$

$$81 + 4K > 0$$

$$4K > -81$$

$$K > -\frac{81}{4}$$ ← first critical value

Second critical value can be found using graph:



K cannot be positive as positive quadratic graph shape (U) will NOT intersect graph 4 times (maximum 2).

K cannot be zero ∵ $y=0$ only intersects $y = \frac{1}{x^2} - 9$ 2 times.

∴ $K < 0$

negative K value allows quadratic to intersect 4 times.

Question 10 continued

$\therefore -\frac{81}{4} < k < 0$

Q10

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

END